MATH 2411 T1B Tutorial 8

HOU Zhen

zhouah@connect.ust.hk

April 14, 2026

Hypothesis Testing — Concepts

The null hypothesis H_0: It is the hypothesis that is assumed to be true and then tested to be rejected or not to be rejected formally.

The alternative hypothesis H_1: It is the hypothesis that typically represents the underlying research question of the investigator and is the complement of H_0.

Test statistic: How we use the data to decide between the two hypotheses.

Decision rule: Based on the value of the test statistic.

Caution: interpreting the decision

We use **the data** only to judge whether there is **enough evidence to reject** H_0.

- If we reject H_0, we have strong grounds to believe H_0 is false and H_1 is true.
- If we **do not have enough evidence to reject** H_0, that **does not** mean we are highly confident that H_0 is true.

Type I and Type II errors

Type I error: We (wrongly) rejected H_0, when H_0 is true. The type I error probability is denoted as $\alpha = P(\text{reject } H_0, \text{ under the condition of } H_0)$.

Type II error: We (wrongly) did not reject H_0, when H_1 is true. The type II error probability is denoted as
$\beta = P(\text{not reject } H_0, \text{ under the condition of } H_1)$.

Methodology: The rejection rule is chosen such that Type I error probability α equals a pre-specified small number, which is called the significance level.

Summary

For $X \sim N(\mu, \sigma^2)$,

Null Hypothesis	Condition	Test Statistics	Alternative Hypothesis	Rejection Rule
$H_0 : \mu = \mu_0$	σ^2 is known	$z_0 = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$	$H_1 : \mu \neq \mu_0$	$\lvert z_0 \rvert > z_{\alpha/2}$
			$H_1 : \mu > \mu_0$	$z_0 > z_\alpha$
			$H_1 : \mu < \mu_0$	$z_0 < -z_\alpha$
$H_0 : \mu = \mu_0$	σ^2 is unknown	$t_0 = \dfrac{\bar{x} - \mu_0}{s_{n-1}/\sqrt{n}}$	$H_1 : \mu \neq \mu_0$	$\lvert t_0 \rvert > t_{n-1,\,\alpha/2}$
			$H_1 : \mu > \mu_0$	$t_0 > t_{n-1,\,\alpha}$
			$H_1 : \mu < \mu_0$	$t_0 < -t_{n-1,\,\alpha}$
$H_0 : \sigma^2 = \sigma_0^2$	μ is unknown	$\chi_0^2 = \dfrac{(n-1)s_{n-1}^2}{\sigma_0^2}$	$H_1 : \sigma^2 > \sigma_0^2$	$\chi_0^2 > \chi_{n-1,\,\alpha}^2$
			$H_1 : \sigma^2 \neq \sigma_0^2$	$\chi_0^2 > \chi_{n-1,\,\alpha/2}^2$ or $\chi_0^2 < \chi_{n-1,\,1-\alpha/2}^2$
			$H_1 : \sigma^2 < \sigma_0^2$	$\chi_0^2 < \chi_{n-1,\,1-\alpha}^2$

R code

Here are R codes for calculating $z_{0.05}$, $t_{9,\,0.05}$ and $\chi^2_{9,\,0.05}$.

```
qnorm(0.05, lower.tail = FALSE)
qt(0.05, df = 9, lower.tail = FALSE)
qchisq(0.05, df = 9, lower.tail = FALSE)
```

or

```
qnorm(1 - 0.05)
qt(1 - 0.05, df = 9)
qchisq(1 - 0.05, df = 9)
```

Exercise

The breaking strength of a fiber used in manufacturing cloth is required to be at least 160 psi. Past experience has indicated that the standard deviation of the breaking strength is 3 psi. A random sample of 40 specimens from a certain batch is tested and the average breaking strength is found to be 159.8 psi. For $\alpha = 0.05$, should this batch be judged acceptable or not?

Exercise

Past experience indicates that the time for high school seniors to complete a standardized test is a normal random variable with a mean of 35 minutes. If a random sample of 20 high school seniors took an average of 33.1 minutes to complete this test with a standard deviation of 4.3 minutes, test the hypothesis at the 0.05 level of significance that $\mu = 35$ minutes against the alternative that $\mu < 35$ minutes.

Exercise

A soft-drink dispensing machine is said to be out of control if the standard deviation of the contents exceeds 15ml. If a random sample of 25 drinks from this machine has a sample standard deviation of 20.3ml, does this indicate at the 0.05 level of significance that the machine is out of control? Assume that the contents are normally distributed.

Type II error and power

Power $= 1 - \beta = 1 - P(\text{not reject } H_0, \text{ under the condition of } H_1)$.

The power calculation does not depend on the data, thus can be done before the experiments (experiment design).

p-value

p-value: The probability of obtaining a test result that is equal or more extreme (in the direction of rejecting H_0) than the actually observed result, under H_0.

Decision rule: If p-value is less than or equal to the significance level, reject H_0; otherwise, do not reject H_0.

Exercise (Discrete)

Andy claims that he can distinguish between green tea and black tea. To validate this claim, we designed an experiment. We provided Andy with four cups of green tea and four cups of black tea to taste, and then asked him to select the four cups that were green tea.

(1) If all 4 cups Andy selected are green tea, can we believe Andy's claim at the 0.05 level of significance?

(2) If there are 3 cups of green tea and 1 cup of black tea among the 4 cups Andy selected, can we believe Andy's claim at the 0.05 level of significance? How about 0.1 level of significance?

Hint: p-value.

Exercise

Consider the one-sided Z test

$$H_0 : \mu = \mu_0 \quad \text{vs.} \quad H_1 : \mu > \mu_0.$$

We know that, in general, the test statement is "Reject H_0 at significance level α if $\bar{X} > \mu_0 + z_\alpha \cdot \frac{\sigma_X}{\sqrt{n}}$". Suppose that we would like the above test to have power $1 - \beta$ when $\mu_X = \mu_1$ for some given $\mu_1 > \mu_0$. Then, how large should the sample size n be? Note: Here, α, β, μ_0, μ_1, σ_X are already known.