MATH 2411 T1B Tutorial 9

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April 21, 2026

Hypothesis Testing

Exercise

Consider the one-sided Z test

$$H_0 : \mu = \mu_0 \quad \text{vs.} \quad H_1 : \mu > \mu_0.$$

We know that, in general, the test statement is "Reject H_0 at significance level α if

$$\bar{X} > \mu_0 + z_\alpha \cdot \frac{\sigma_X}{\sqrt{n}}.$$

" Suppose that we would like the above test to have power $1 - \beta$ when $\mu_X = \mu_1$ for some given $\mu_1 > \mu_0$. Then, how large should the sample size n be?

Note: Here, $\alpha, \beta, \mu_0, \mu_1, \sigma_X$ are already known.

Exercise

Suppose we have a coin and we want to know whether it is fair. Denote θ as the probability of HEAD. We tossed this coin 100 times and got 40 heads. Consider the hypothesis testing

$$H_0 : \theta = 0.5 \quad \text{vs.} \quad H_1 : \theta < 0.5.$$

1. Find the p-value.
2. Use Central Limit Theorem to find an approximation of p-value.
3. Should we reject null hypothesis at significance level 0.05? How about significance level 0.01?

Analysis of Variance

Two-sample Testing Summary

$X \sim N(\mu_X, \sigma_X^2)$, n iid samples; $Y \sim N(\mu_Y, \sigma_Y^2)$, m iid samples. X_i and Y_j are independent.

Null H_0	Condition	Test Statistic	Alternative H_1	Rejection Rule
$\mu_X = \mu_Y$	σ_X^2, σ_Y^2 known	$Z_0 = \dfrac{\bar{X} - \bar{Y}}{\sqrt{\sigma_X^2/n + \sigma_Y^2/m}}$	$\mu_X \neq \mu_Y$	$\lvert z_0 \rvert > z_{\alpha/2}$
			$\mu_X > \mu_Y$	$z_0 > z_\alpha$
			$\mu_X < \mu_Y$	$z_0 < -z_\alpha$
$\mu_X = \mu_Y$	$\sigma_X^2 = \sigma_Y^2 = \sigma^2$ unknown	$T_0 = \dfrac{\bar{X} - \bar{Y}}{S_p\sqrt{1/n + 1/m}}$	$\mu_X \neq \mu_Y$	$\lvert t_0 \rvert > t_{n+m-2,\alpha/2}$
			$\mu_X > \mu_Y$	$t_0 > t_{n+m-2,\alpha}$
			$\mu_X < \mu_Y$	$t_0 < -t_{n+m-2,\alpha}$
$\mu_X = \mu_Y$	σ_X^2, σ_Y^2 unknown, unequal	$T_0 = \dfrac{\bar{X} - \bar{Y}}{\sqrt{S_{n-1,X}^2/n + S_{m-1,Y}^2/m}}$	$\mu_X \neq \mu_Y$	$\lvert t_0 \rvert > t_{\nu,\alpha/2}$
			$\mu_X > \mu_Y$	$t_0 > t_{\nu,\alpha}$
			$\mu_X < \mu_Y$	$t_0 < -t_{\nu,\alpha}$

where

$$S_p^2 = \frac{(n-1)S_{n-1,X}^2 + (m-1)S_{m-1,Y}^2}{n+m-2},$$

and

$$\nu = \frac{\left(\dfrac{s_{n-1,X}^2}{n} + \dfrac{s_{m-1,Y}^2}{m}\right)^2}{\dfrac{1}{n-1}\left(\dfrac{s_{n-1,X}^2}{n}\right)^2 + \dfrac{1}{m-1}\left(\dfrac{s_{m-1,Y}^2}{m}\right)^2}.$$

Exercise

Problem: Comparing Battery Life of Two Brands
A consumer research group compares two AA battery brands under identical continuous-use conditions.

Data Collected
Brand A ($n_1 = 12$): $7.3, 8.1, 7.8, 7.5, 8.0, 7.6, 8.2, 7.9, 7.7, 8.3, 7.4, 8.0$;
$\bar{x}_1 = 7.817$, $s_1 = 0.321$.
Brand B ($n_2 = 15$):
$7.1, 7.4, 6.9, 7.3, 7.6, 7.0, 7.2, 7.5, 6.8, 7.3, 7.1, 7.4, 7.0, 7.2, 7.3$; $\bar{x}_2 = 7.207$, $s_2 = 0.225$.

At significance level $\alpha = 0.05$: Assume the two populations have equal variances. Perform a two-sided pooled two-sample t-test:

$$H_0 : \mu_1 = \mu_2 \qquad \text{vs.} \qquad H_1 : \mu_1 \neq \mu_2.$$

Compute the test statistic, degrees of freedom, and p-value, then state your conclusion.

Exercise

A university facilities manager wants to compare the mean cup volumes dispensed by two coffee machines:

$X \sim N(\mu_X, \sigma_X^2)$ (Math building), $\qquad Y \sim N(\mu_Y, \sigma_Y^2)$ (Engineering building)

Both machines are known to have variance

$$\sigma_X^2 = \sigma_Y^2 = 5 \text{ mL}^2.$$

The manager plans to collect n cups from each machine and conduct a one-sided z-test

$$H_0 : \mu_X = \mu_Y \quad \text{vs.} \quad H_1 : \mu_X > \mu_Y$$

at significance level $\alpha = 0.05$.

Determine the minimum equal sample size $n = m$ needed to detect a true mean difference

$$\mu_X - \mu_Y = 3 \text{ mL}$$

with power $1 - \beta = 0.90$.