MATH 3033 T2A Tutorial 8

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Problem Set 7

4. Let f is continuous on $[0, 1]$ and $f(1) = 0$. Define $g_n(x) = f(x)x^n$. Show that $\{g_n\}$ uniformly converges on $[0, 1]$.

Problem Set 8

1. Discuss the uniform convergence of the sequence of functions

$$f_n(x) = \frac{nx}{1+n^2x^2}$$

on the intervals $(0, +\infty)$ and $[\delta, +\infty)$, where $\delta > 0$.

Hint: Show $||f_n(x) - f(x)||_{(0,\infty)} \not\to 0$ and $||f_n(x) - f(x)||_{[\delta,+\infty)} \to 0$.

2. Discuss the uniform convergence of the sequence of functions

$$f_n(x) = 2n^2xe^{-n^2x^2}, \quad (n = 1, 2, ...)$$

on the interval $[0, 1]$.

Hint: Show $||f_n(x) - f(x)||_{[0,1]} \to 0$.

3. Discuss the uniform convergence of the series

$$\sum_{n=1}^{\infty} ne^{-nx}$$

on the interval $(0, +\infty)$.

Hint: Show ne^{-nx} doesn't converge uniformly to 0 on $(0, +\infty)$.

4. Let $f(x) = \sum_{n=0}^{\infty} \dfrac{x^n}{3^n} \cos n\pi x^2$. Calculate $\lim_{x\to 1} f(x)$.

   Hint: Show $f(x)$ uniformly converges on $[-2, 2]$.