MATH 3033 Tutorial Problem Set 1

1. (Parallelogram Law) Prove that for any vectors $u, v \in \mathbb{R}^n$, we have

$$\|u+v\|^2 + \|u-v\|^2 = 2\|u\|^2 + 2\|v\|^2.$$

Remark: This property can be generalized to any **inner product space**.

Remark: An interesting question is: Is this property also true for any **norm vector space**? If not, for what kind of normed space is this property true?

2. (1) Prove that for any two sets A and B, we have $\overline{A \cup B} = \overline{A} \cup \overline{B}$.
   (2) Find such sets A and B that $\overline{A \cap B} \neq \overline{A} \cap \overline{B}$.

3. Assume $S \subseteq \mathbb{R}$. Prove that if S' is countable, then S is also countable.

4. Define the addition between two sets S_1 and S_2 as $S_1 + S_2 = \{x + y : x \in S_1, y \in S_2\}$.
   Assume that A_1 and A_2 are two non-empty subset of $\mathbb{R}$ and $A_1' \neq \emptyset$.
   Prove that $\overline{A}_1 + A_2' \subseteq (A_1 + A_2)'$.