

MATH 3033 Tutorial Problem Set 9

1. Find the convergence region of each below power series.

(1) $\sum_{n=1}^{\infty} (nx)^n;$

(2) $\sum_{n=1}^{\infty} \frac{x^n}{n!};$

(3) $\sum_{n=1}^{\infty} x^n, \sum_{n=1}^{\infty} \frac{x^n}{n}, \sum_{n=1}^{\infty} \frac{x^n}{n^2}.$

2. Consider the following power series centered at 0:

$$\sum_{n=0}^{\infty} a_n x^n = 1 + \frac{1}{5}x + 2x^2 + \frac{1}{5^2}x^3 + 3x^4 + \frac{1}{5^3}x^5 + 4x^6 + \dots$$

- (1) Write down the formula for the general term a_n .

- (2) Compute $\limsup_{n \rightarrow \infty} |a_n|^{\frac{1}{n}}$ and hence the radius of convergence of the power series.

3. Write the Taylor series of functions below centered at $x = 0$ and then specify the interval of convergence.

(1) $f(x) = \cos^2 x.$

(2) $f(x) = \ln(1 - x - 2x^2).$

4. Calculate $\sum_{n=1}^{\infty} \frac{n(n+1)}{(2n)!!}.$